

***Project Management Finance and
Risk
PERT and CPM***

The Project

A project is a temporary endeavour involving a connected sequence of activities and a range of resources, which is designed to achieve a specific and unique outcome, and which operates within time, cost and quality constraints and which is often used to introduce change.

Characteristic of a project

- A unique, one-time operational activity or effort
- Requires the completion of many interrelated activities
- Established to achieve specific objective
- Resources, such as time and/or money, are limited
- Typically has its own management structure
- Need leadership

Project Examples

- constructing houses, factories, shopping malls, athletic stadiums or arenas
- developing military weapons systems, aircrafts, new ships
- launching satellite systems
- constructing oil pipelines
- developing and implementing new computer systems
- planning concert, football games, or basketball tournaments
- introducing new products into market

What is project management

- The application of a collection of tools and techniques to direct the use of diverse resources towards the accomplishment of a unique, complex, one-time task within time, cost and quality constraints.
- Its origins lie in World War II, when the military authorities used the techniques of operational research to plan the optimum use of resources.
- One of these techniques was the use of networks to represent a system of related activities

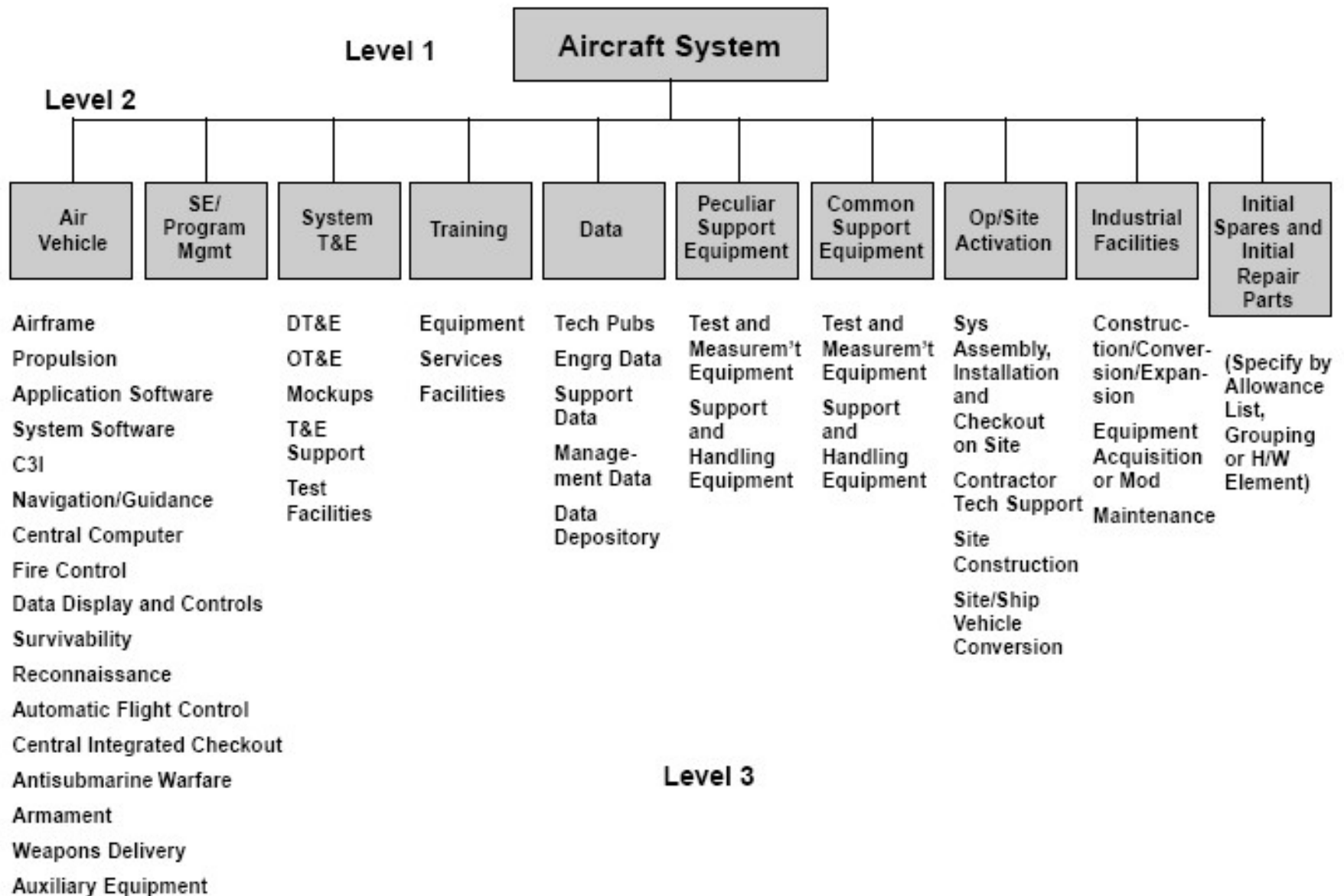
Project Management Process

- Project planning
- Project scheduling
- Project control
- Project team - made up of individuals from various areas and departments within a company
- Matrix organization - a team structure with members from functional areas, depending on skills required
- Project Manager - leader of the project team
- Scope statement - a document that provides an understanding, justification, and expected result of a project
- Statement of work - written description of objectives of a project
- Organizational Breakdown Structure - a chart that shows which organizational units are responsible for work items
- Responsibility Assignment Matrix - shows who is responsible for work in a project

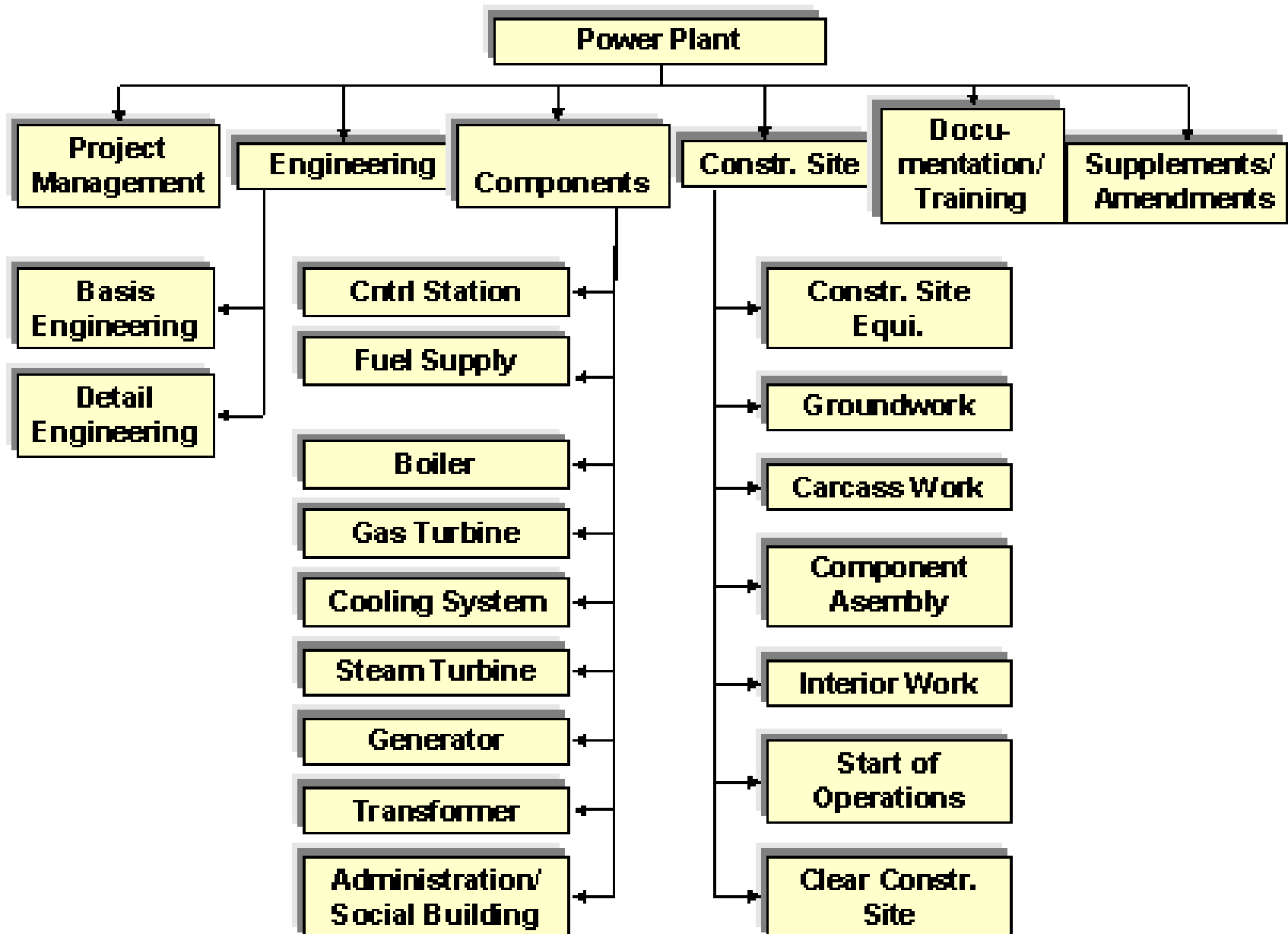
Work breakdown structure

- A method of breaking down a project into individual elements (components, subcomponents, activities and tasks) in a hierarchical structure which can be scheduled and cost
- It defines tasks that can be completed independently of other tasks, facilitating resource allocation, assignment of responsibilities and measurement and control of the project
- It is foundation of project planning
- It is developed before identification of dependencies and estimation of activity durations
- It is used to identify the tasks in the CPM and PERT

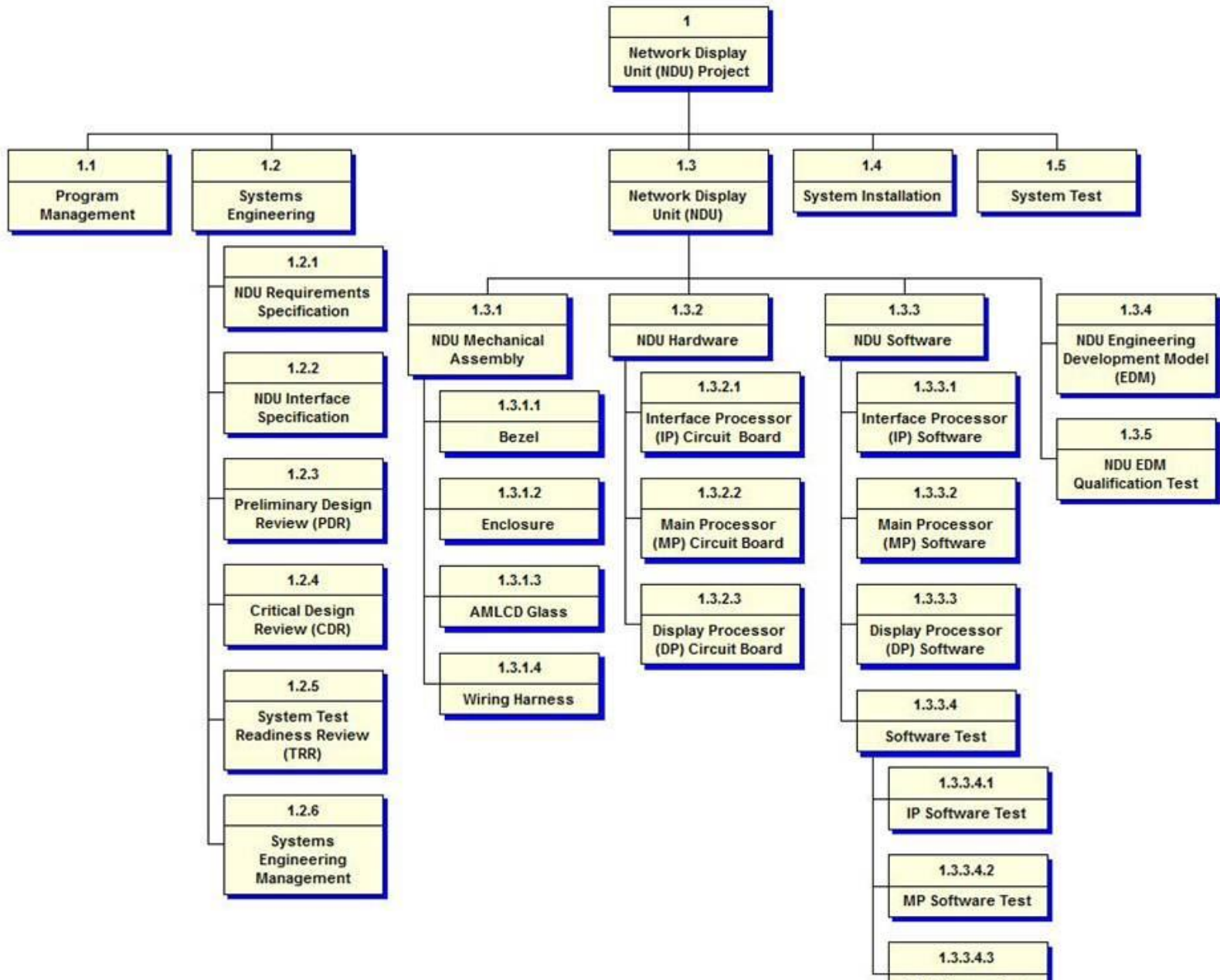
Example WBS System level



Plant level



Network level



Project Planning

- Resource Availability and/or Limits
 - Due date, late penalties, early completion incentives
 - Budget
- Activity Information
 - Identify all required activities
 - Estimate the resources required (time) to complete each activity
 - Immediate predecessor(s) to each activity needed to create interrelationships

Project Scheduling & Control Techniques

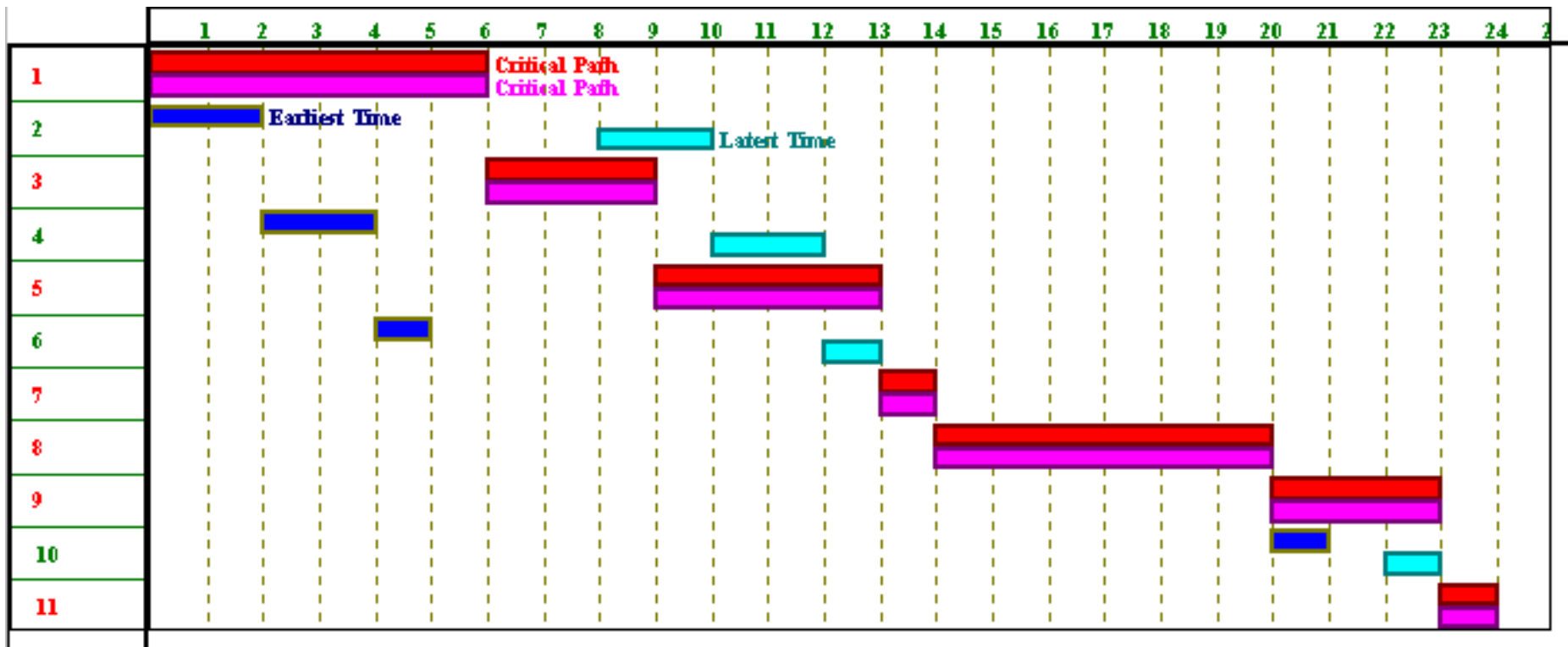
Gantt Chart

Critical Path Method (CPM)

Program Evaluation and Review Technique (PERT)

Gantt Chart

- ♦ Graph or bar chart with a bar for each project activity that shows passage of time
- ♦ Provides visual display of project schedule



History of CPM/PERT

- Critical Path Method (CPM)
 - E I Du Pont de Nemours & Co. (1957) for construction of new chemical plant and maintenance shut-down
 - Deterministic task times
 - Activity-on-node network construction
 - Repetitive nature of jobs
- Project Evaluation and Review Technique (PERT)
 - U S Navy (1958) for the POLARIS missile program
 - Multiple task time estimates (probabilistic nature)
 - Activity-on-arrow network construction
 - Non-repetitive jobs (R & D work)

Project Network

Network analysis is the general name given to certain specific techniques which can be used for the planning, management and control of projects

- Use of nodes and arrows

Arrows □ An arrow leads from tail to head directionally. Indicates ACTIVITY, a time consuming effort required to perform a part of the work.

Nodes ● A node is represented by a circle. Indicates an EVENT, a point in time where one or more activities start and/or finish.

- Activity
 - A task or a certain amount of work required in the project
 - Requires time to complete
 - Represented by an arrow
- Dummy Activity
 - Indicates only precedence relationships

Project Network

- Event
 - Signals the beginning or ending of an activity
 - Designates a point in time
 - Represented by a circle (node)
- Network
 - Shows the sequential relationships among activities using nodes and arrows

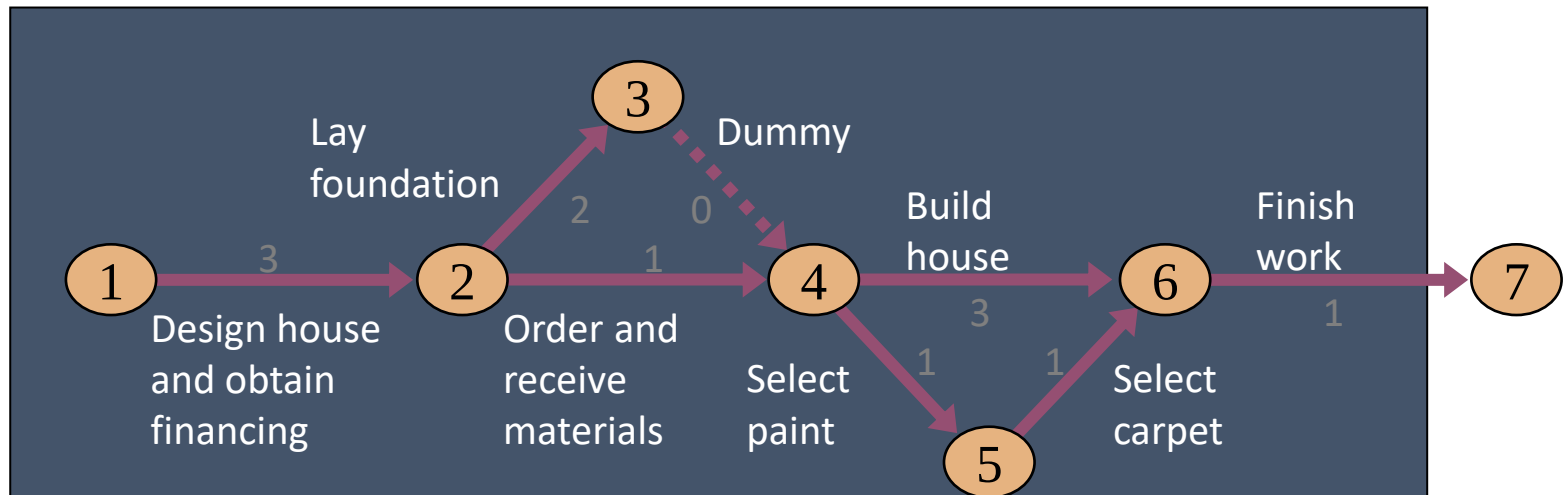
❖ Activity-on-node (AON)

- ❖ nodes represent activities, and arrows show precedence relationships

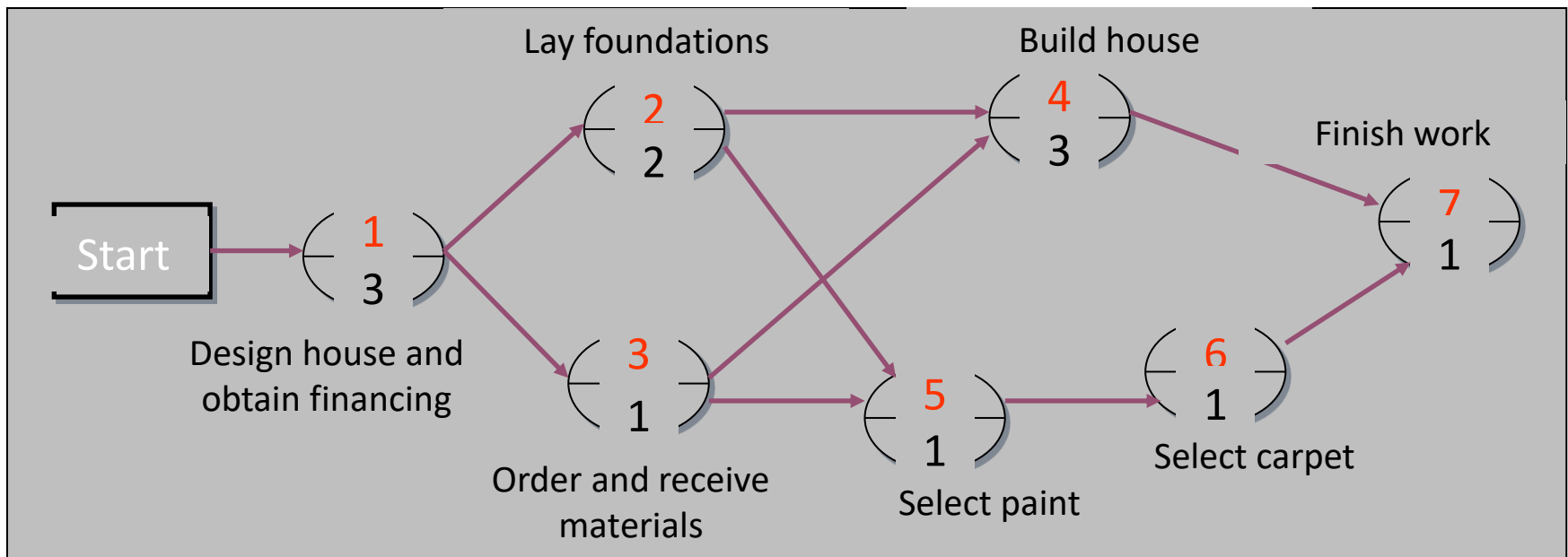
❖ Activity-on-arrow (AOA)

- ❖ arrows represent activities and nodes are events for points in time

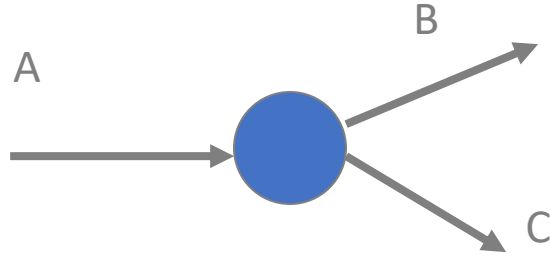
AOA Project Network for House



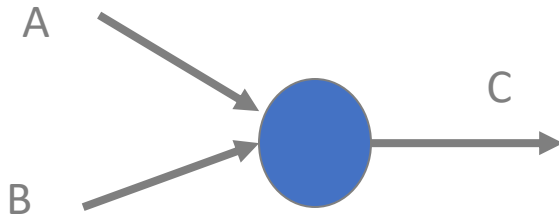
AON Project Network for House



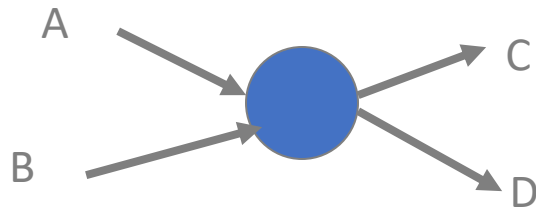
Situations in network diagram



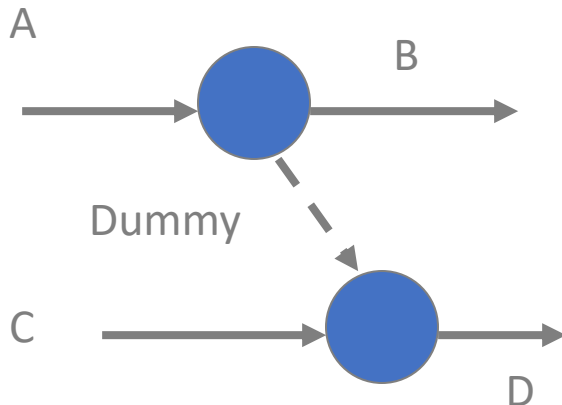
A must finish before either B or C can start



both A and B must finish before C can start



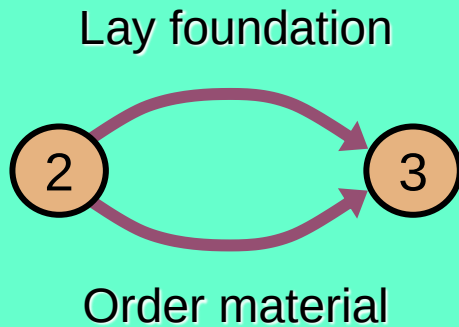
both A and B must finish before either of C or D can start



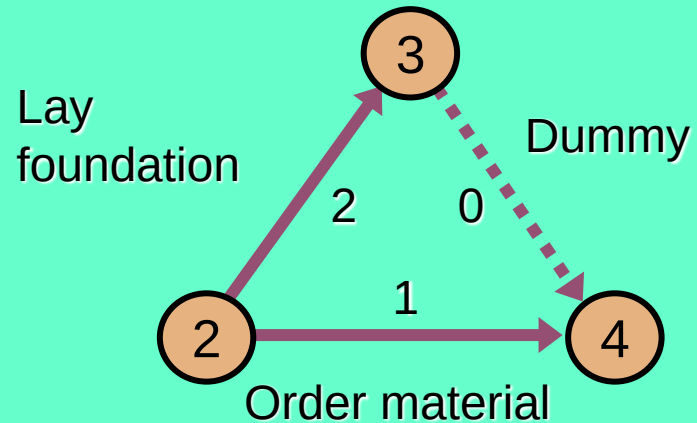
A must finish before B can start

both A and C must finish before D can start

Concurrent Activities



(a) Incorrect precedence relationship



(b) Correct precedence relationship

Network example

How long will it take to complete this project ?

Illustration of network analysis of a minor redesign of a product and its associated packaging.

Activity number		Completion time (weeks)
1	Redesign product	6
2	Redesign packaging	2
3	Order and receive components for redesigned product	3
4	Order and receive material for redesigned packaging	2
5	Assemble products	4
6	Make up packaging	1
7	Package redesigned product	1
8	Test market redesigned product	6
9	Revise redesigned product	3
10	Revise redesigned packaging	1
11	Present results to the Board	1

Network example

For clarity, this list is kept to a minimum by specifying only immediate relationships, that is relationships involving activities that "occur near to each other in time".

Activity
number

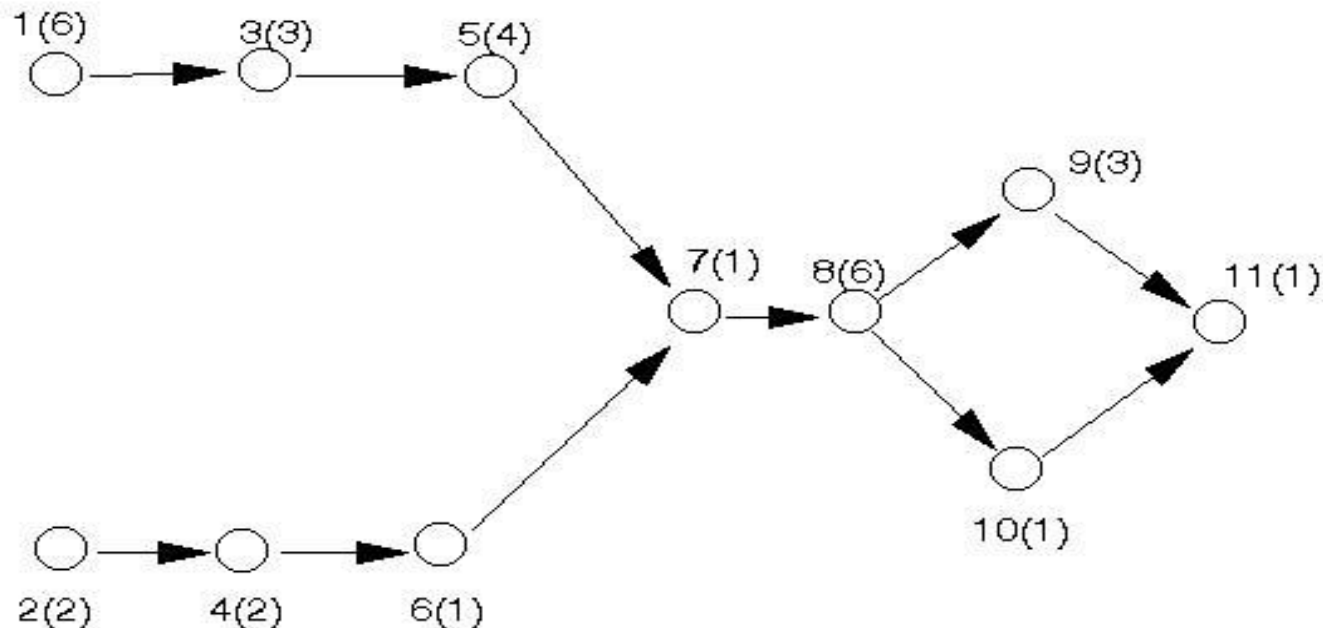
1 must be finished before
2
3
4
5, 6
7
8
8
9, 10

Activity
number

3 can start
4
5
6
7
8
9
10
11

Preparing the activity network

- Is this a Start Activity?
- Is this a Finish Activity?
- What Activity Precedes this?
- What Activity Follows this?
- What Activity is Concurrent with this?



CPM calculation

- Path
 - A connected sequence of activities leading from the starting event to the ending event
- Critical Path
 - The longest path (time); determines the project duration
- Critical Activities
 - All of the activities that make up the critical path

Forward Pass

- Earliest Start Time (ES)
 - earliest time an activity can start
 - $ES = \text{maximum EF of immediate predecessors}$
- Earliest finish time (EF)
 - earliest time an activity can finish
 - earliest start time plus activity time: **$EF = ES + t$**

Backward Pass

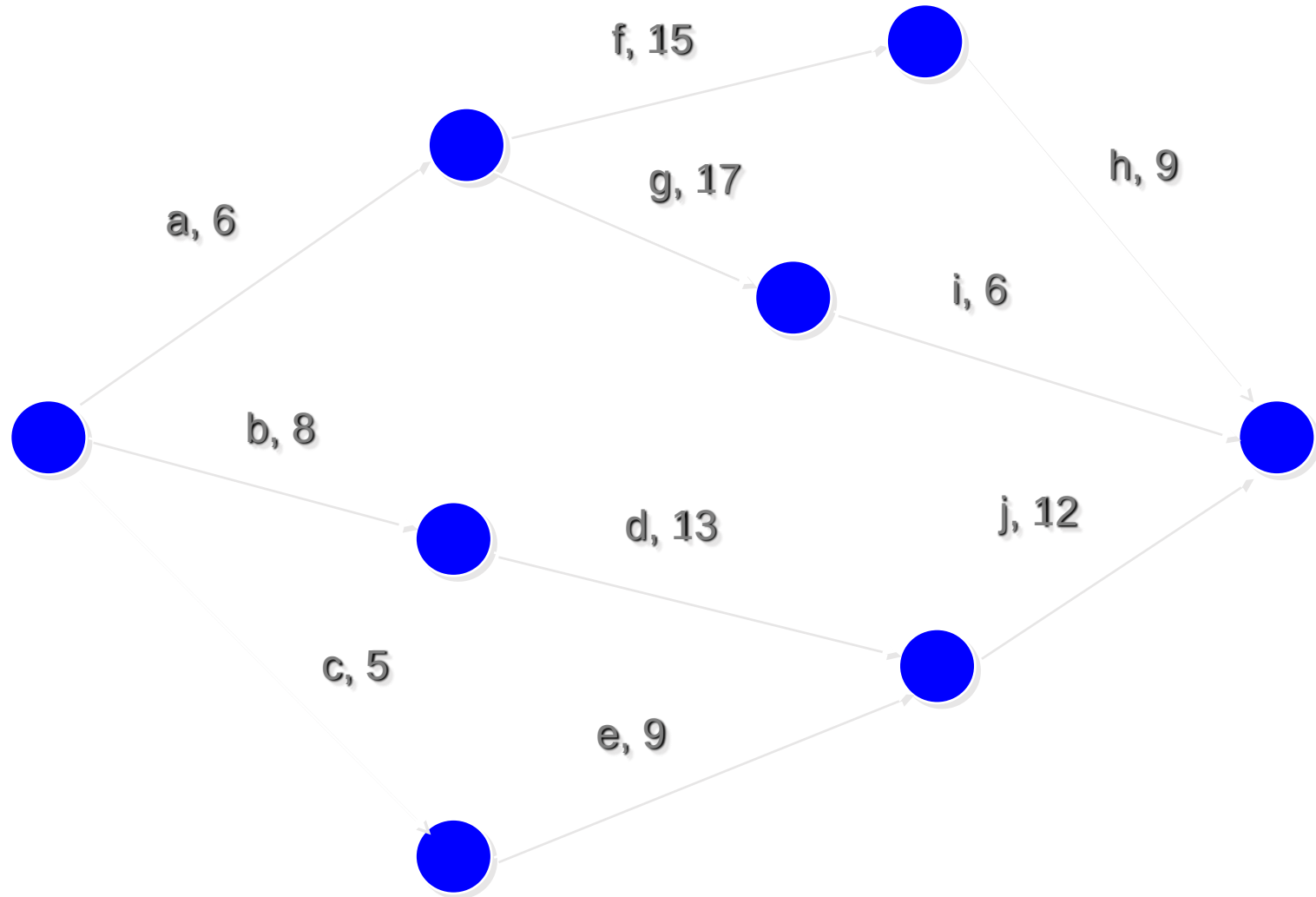
- Latest Start Time (LS)
 - Latest time an activity can start without delaying critical path time: $LS = LF - t$
- Latest finish time (LF)
 - latest time an activity can be completed without delaying critical path time
 - $LS = \text{minimum LS of immediate predecessors}$

CPM analysis

- Draw the CPM network
- Analyze the paths through the network
- Determine the float for each activity
 - Compute the activity's float
$$\text{float} = \text{LS} - \text{ES} = \text{LF} - \text{EF}$$
 - Float is the maximum amount of time that this activity can be delay in its completion before it becomes a critical activity, i.e., delays completion of the project
- Find the critical path is that the sequence of activities and events where there is no “slack” i.e.. ***Zero slack***
 - Longest path through a network
- Find the project duration is minimum project completion time

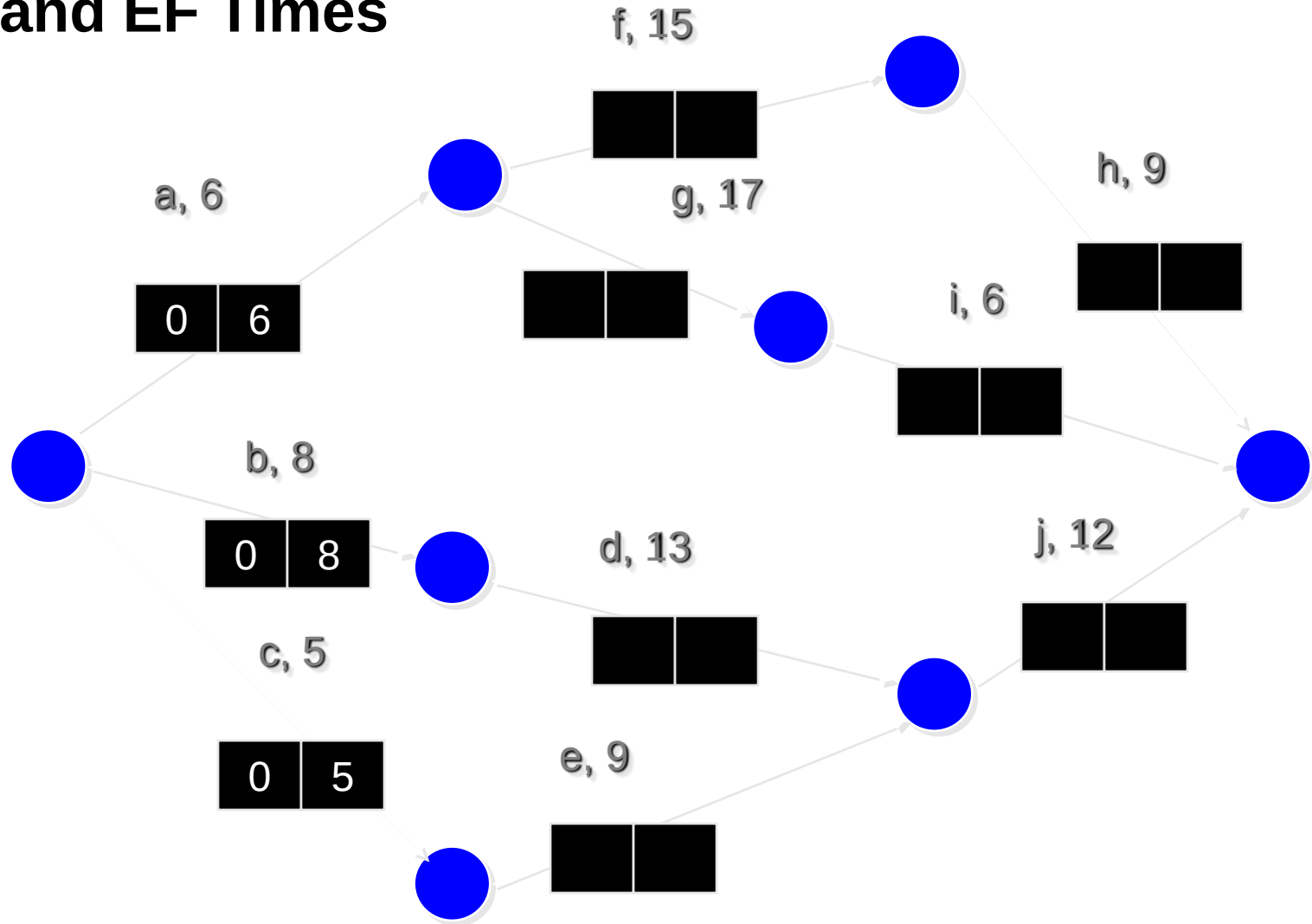
CPM Example:

CPM Network



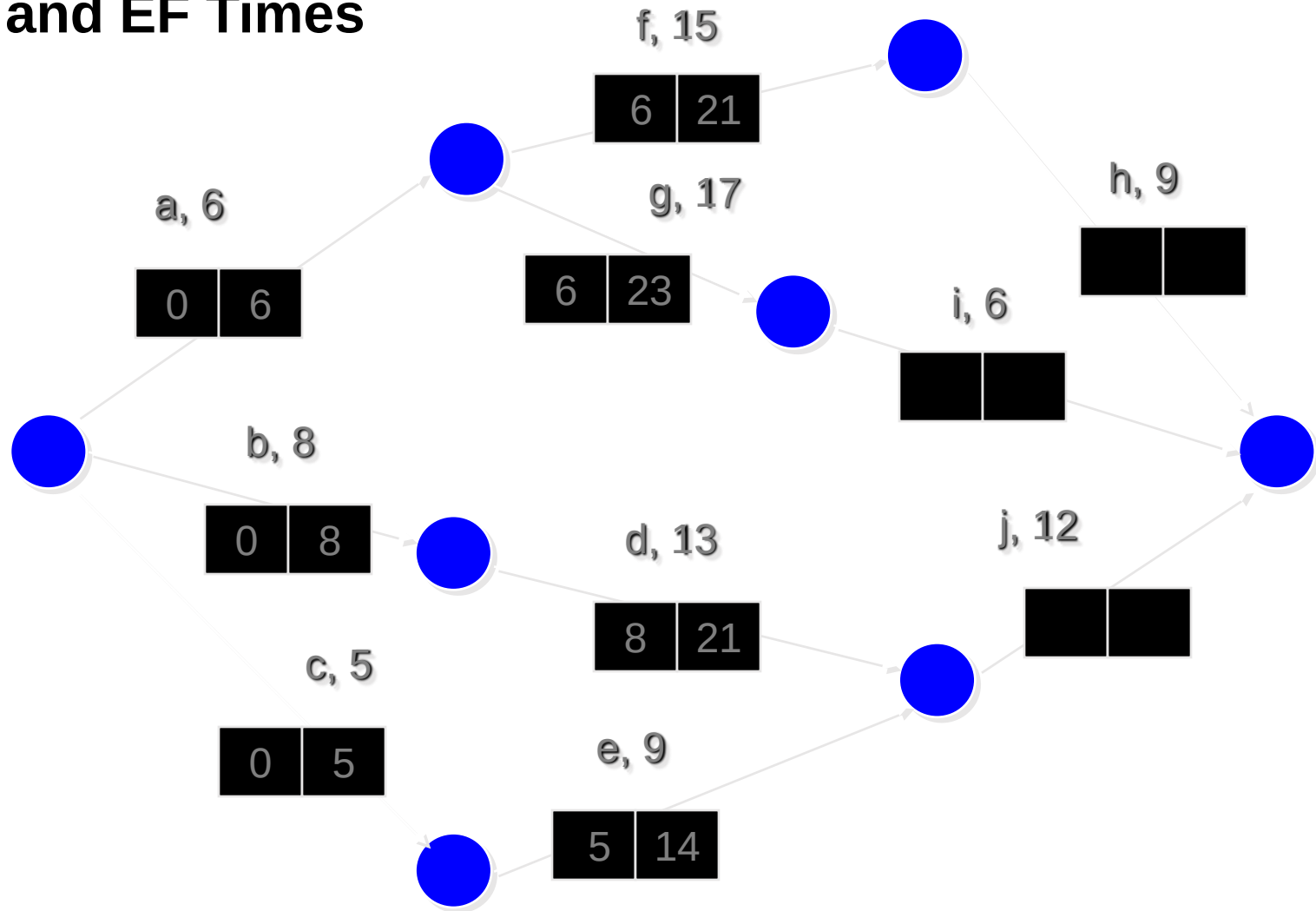
CPM Example

ES and EF Times



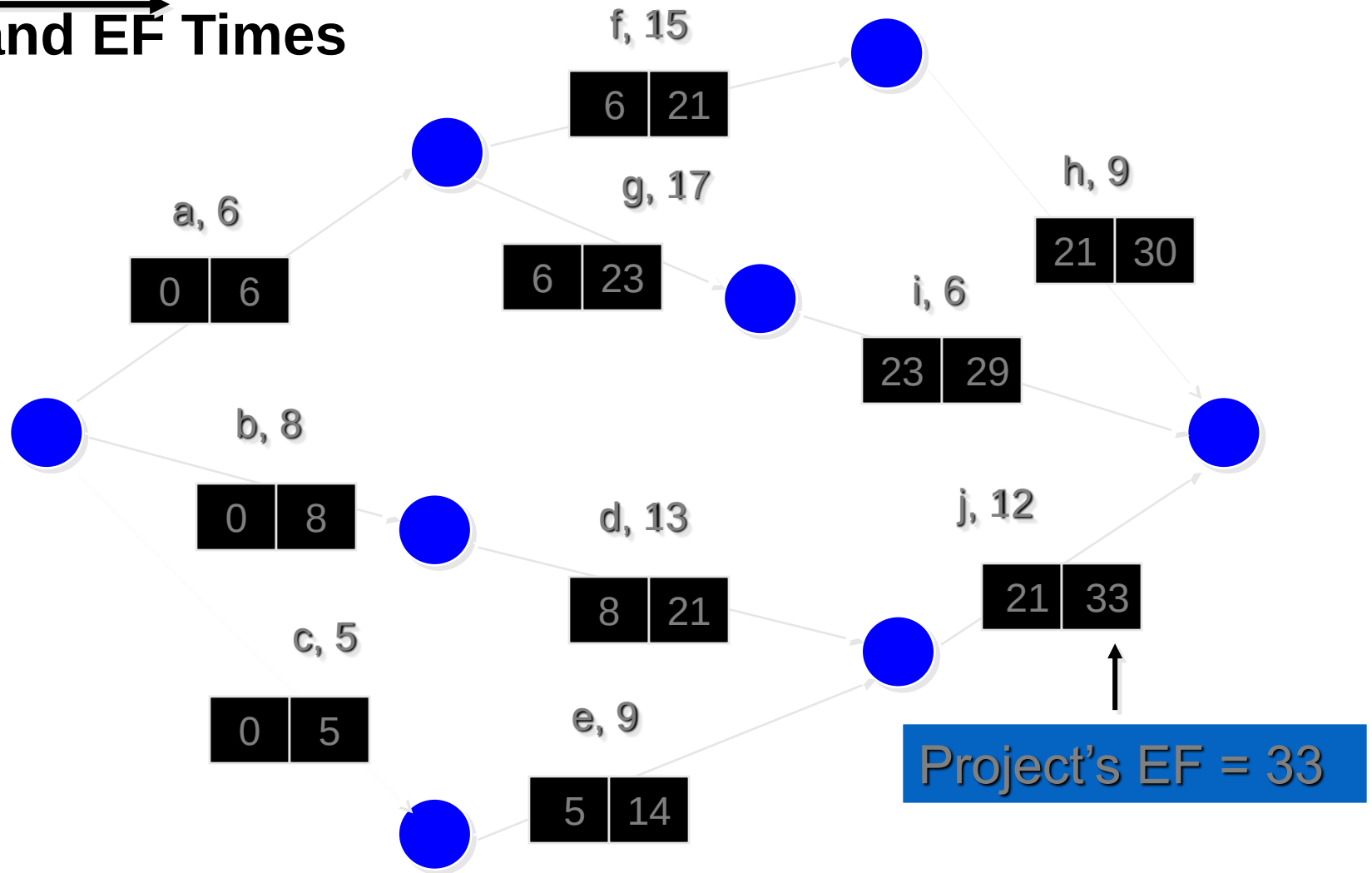
CPM Example

→
ES and EF Times



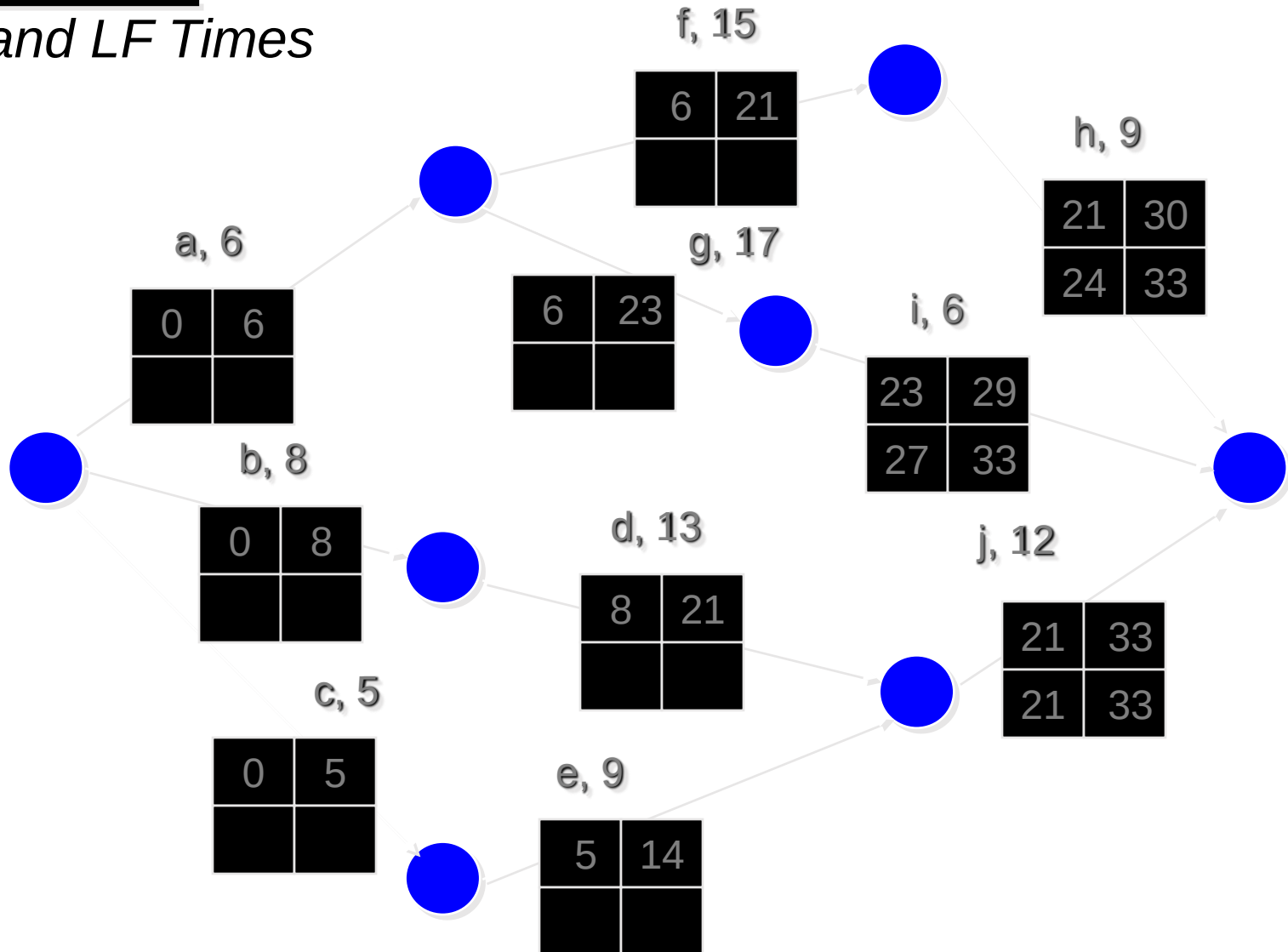
CPM Example

ES and EF Times



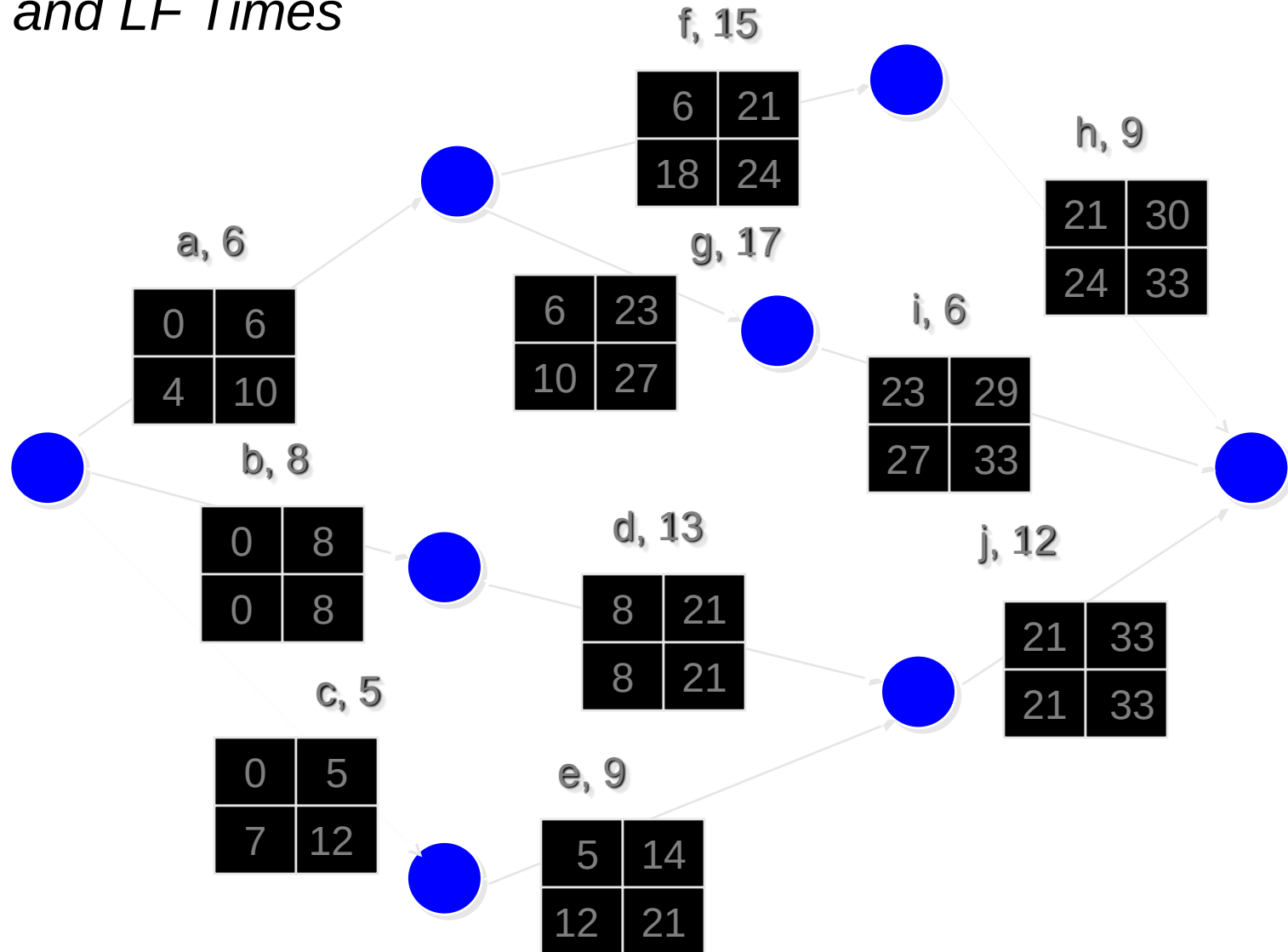
CPM Example

←
LS and LF Times



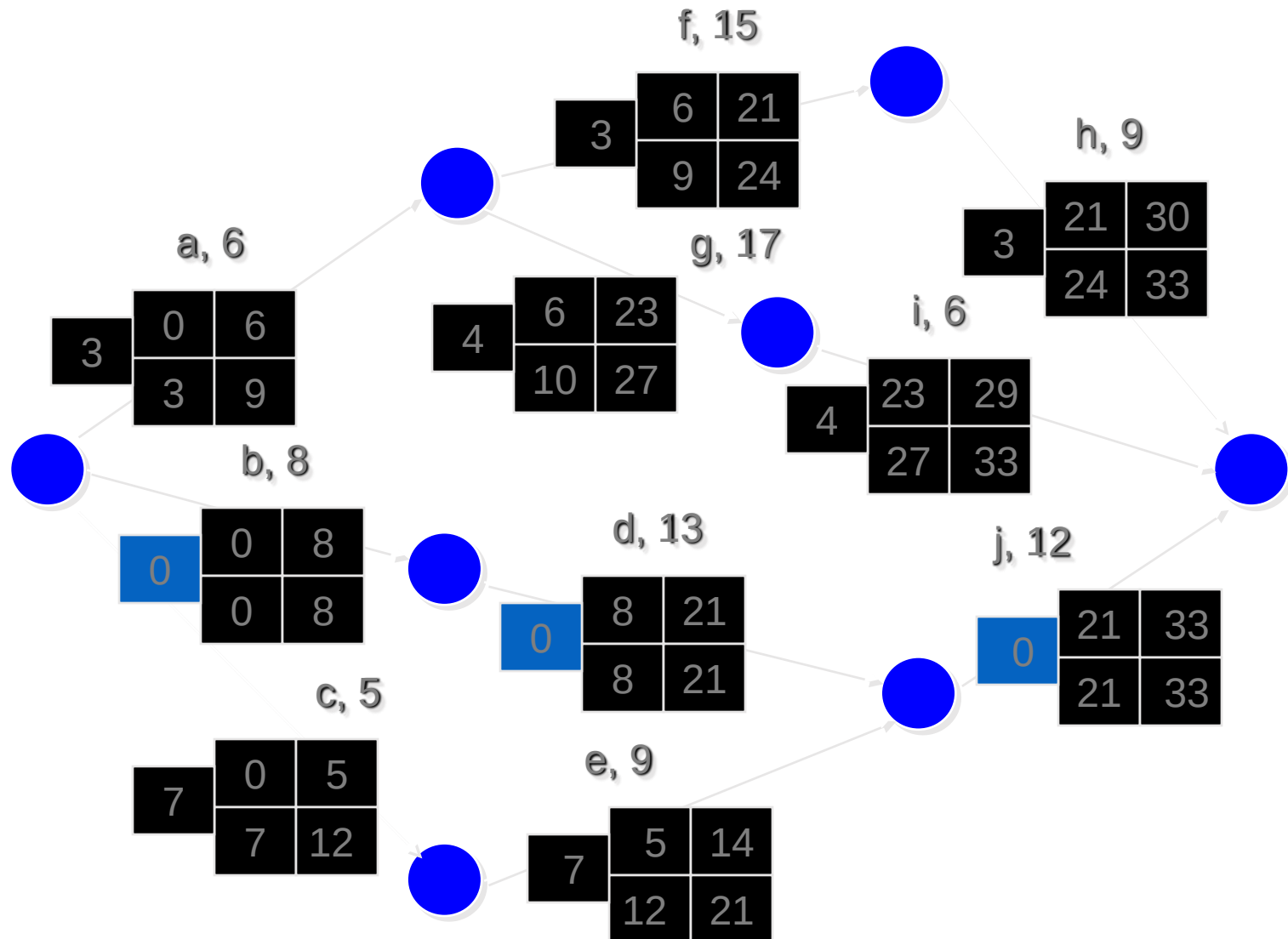
CPM Example

←
LS and LF Times



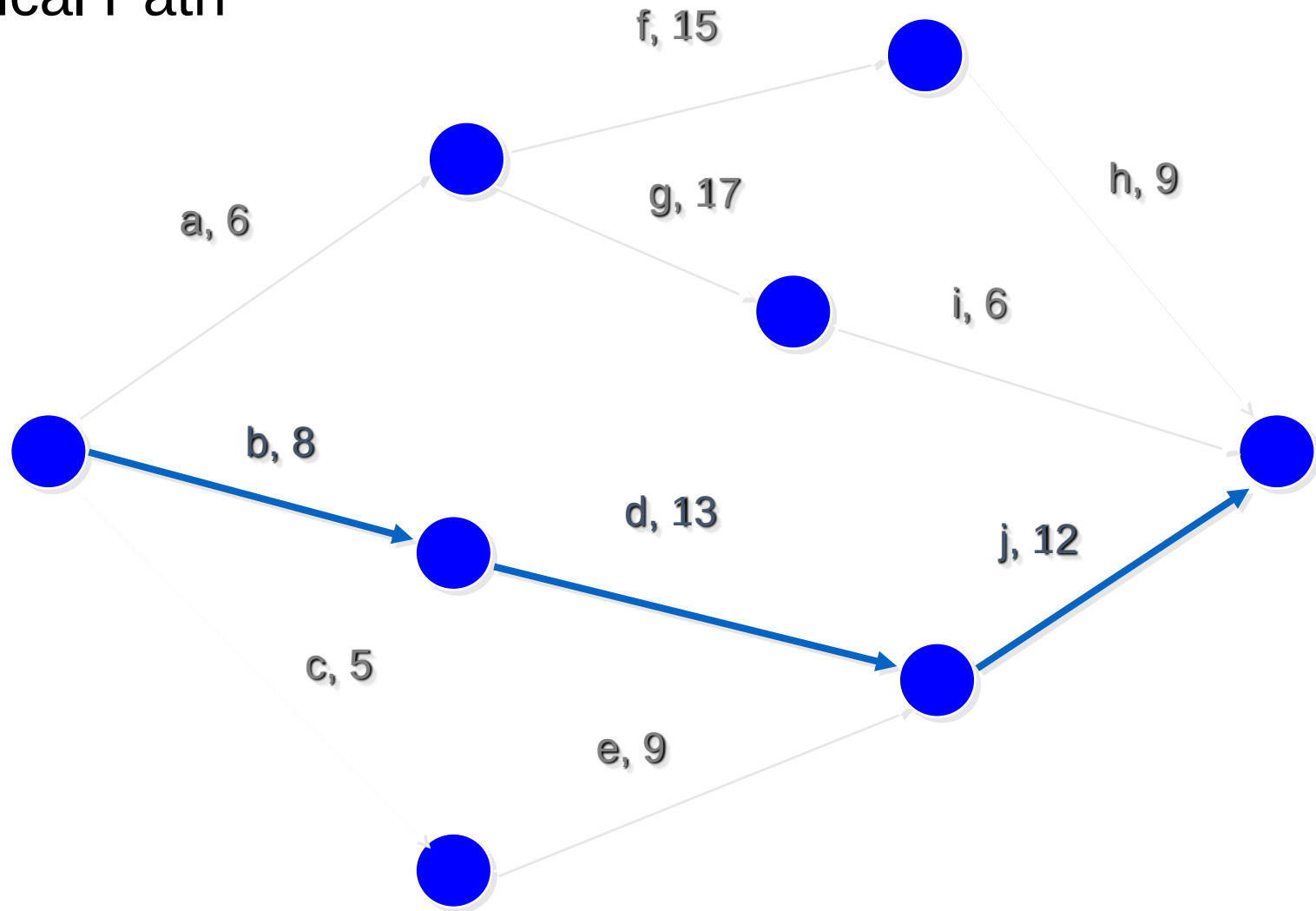
CPM Example

- Float



CPM Example

Critical Path



PERT

- PERT assumes that an activity's duration follows a probability distribution instead of being a single value
- Three time estimates are required to compute the parameters of an activity's duration distribution:
 - pessimistic time (t_p) - the time the activity would take if things did not go well
 - most likely time (t_m) - the consensus best estimate of the activity's duration
 - optimistic time (t_o) - the time the activity would take if things did go well

Mean (expected time): $t_e = \frac{t_p + 4 t_m + t_o}{6}$

Variance: $V_t = \sigma^2 = \left(\frac{t_p - t_o}{6} \right)^2$

PERT analysis

- Draw the network.
- Analyze the paths through the network and find the critical path.
- The length of the critical path is the mean of the project duration probability distribution which is assumed to be normal
- The standard deviation of the project duration probability distribution is computed by adding the variances of the critical activities (all the activities that make up the critical path) and taking the square root of that sum
- Probability computations can now be made using the normal distribution table.

Probability computation

Determine probability that project is completed within specified time

$$Z = \frac{x - \mu}{\sigma}$$

where $\mu = t_p$ = project mean time

σ = project standard mean time

x = (proposed) specified time

Normal Distribution - Project Time

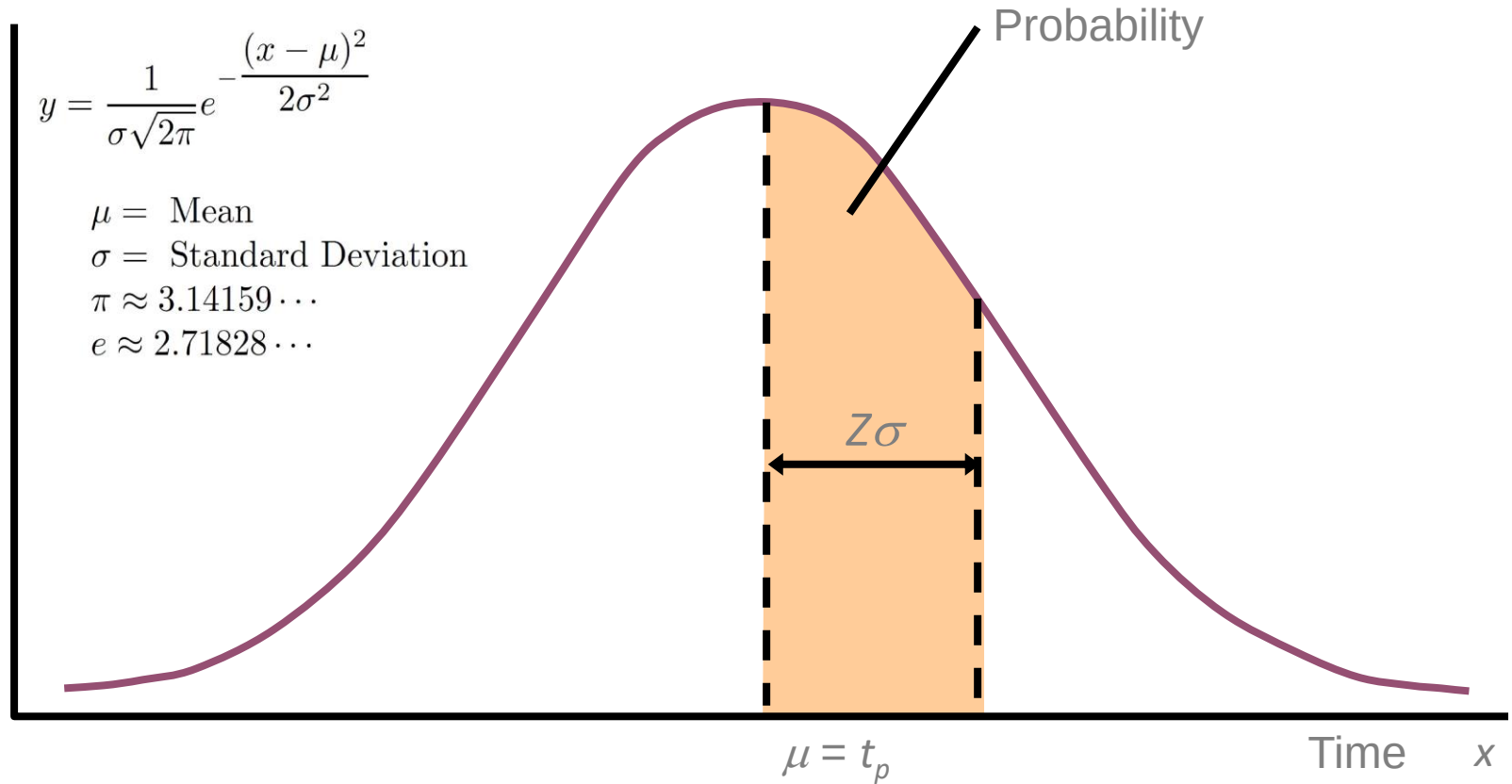
$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}}$$

μ = Mean

σ = Standard Deviation

$\pi \approx 3.14159\dots$

$e \approx 2.71828\dots$

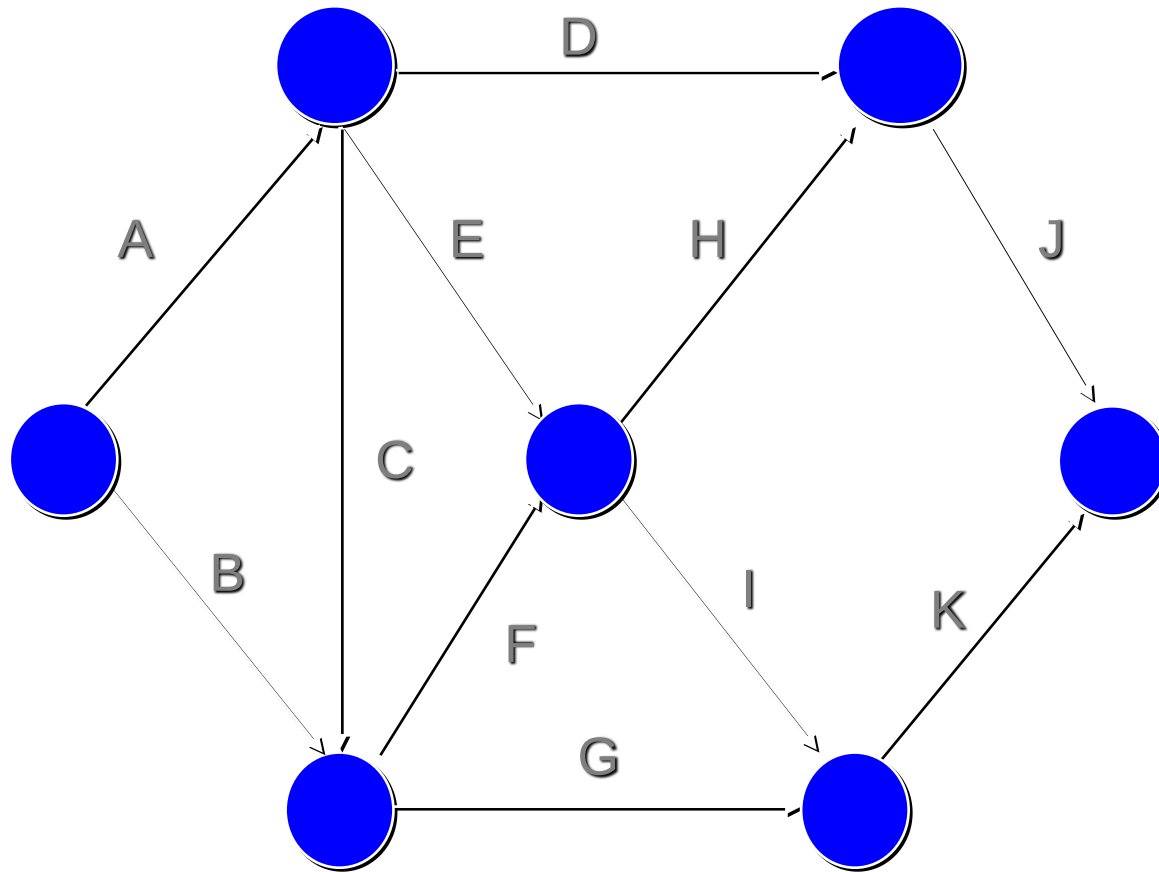


PERT Example

	Immed.	Optimistic	Most Likely	Pessimistic
<u>Activity</u>	<u>Predec.</u>	<u>Time (Hr.)</u>	<u>Time (Hr.)</u>	<u>Time (Hr.)</u>
A	--	4	6	8
B	--	1	4.5	5
C	A	3	3	3
D	A	4	5	6
E	A	0.5	1	1.5
F	B,C	3	4	5
G	B,C	1	1.5	5
H	E,F	5	6	7
I	E,F	2	5	8
J	D,H	2.5	2.75	4.5
K	G,I	3	5	7

PERT Example

PERT Network



PERT Example

<u>Activity</u>	<u>Expected Time</u>	<u>Variance</u>
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

PERT Example

<u>Activity</u>	<u>ES</u>	<u>EF</u>	<u>LS</u>	<u>LF</u>	<u>Slack</u>
A	0	6	0	6	0 *
B	0	4	5	9	5
C	6	9	6	9	0 *
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0 *
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0 *
J	19	22	20	23	1
K	18	23	18	23	0 *

* Critical

PERT Example

$$\begin{aligned}V_{\text{path}} &= V_A + V_C + V_F + V_I + V_K \\&= 4/9 + 0 + 1/9 + 1 + 4/9 \\&= 2\end{aligned}$$

$$\sigma_{\text{path}} = 1.414$$

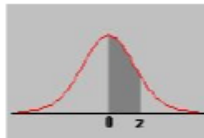
$$z = (24 - 23)/\sigma = (24-23)/1.414 = .71$$

From the Standard Normal Distribution table:

$$P(z \leq .71) = .5 + .2612 = .7612$$

There is a 76.12% chance of this project being completed within the time period stated .

Standard Normal (Z) Table
Area between 0 and z



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

PROJECT COST

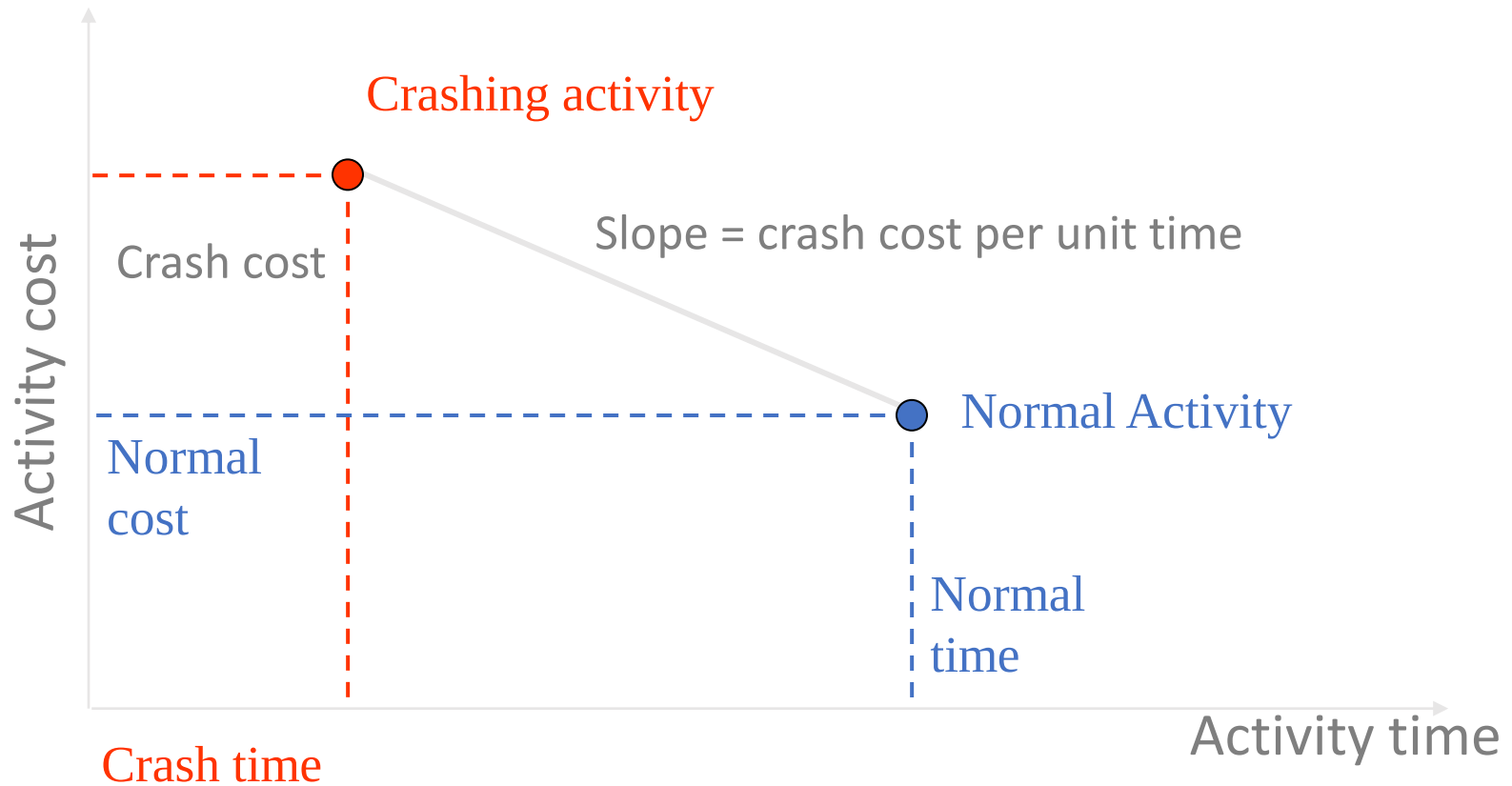
Cost consideration in project

- Project managers may have the option or requirement to crash the project or accelerate the completion of the project.
- This is accomplished by reducing the length of the critical path(s).
- The length of the critical path is reduced by reducing the duration of the activities on the critical path.
- If each activity requires the expenditure of an amount of money to reduce its duration by one unit of time, then the project manager selects the least cost critical activity, reduces it by one time unit, and traces that change through the remainder of the network.
- As a result of a reduction in an activity's time, a new critical path may be created.
- When there is more than one critical path, each of the critical paths must be reduced.
- If the length of the project needs to be reduced further, the process is repeated.

Project Crashing

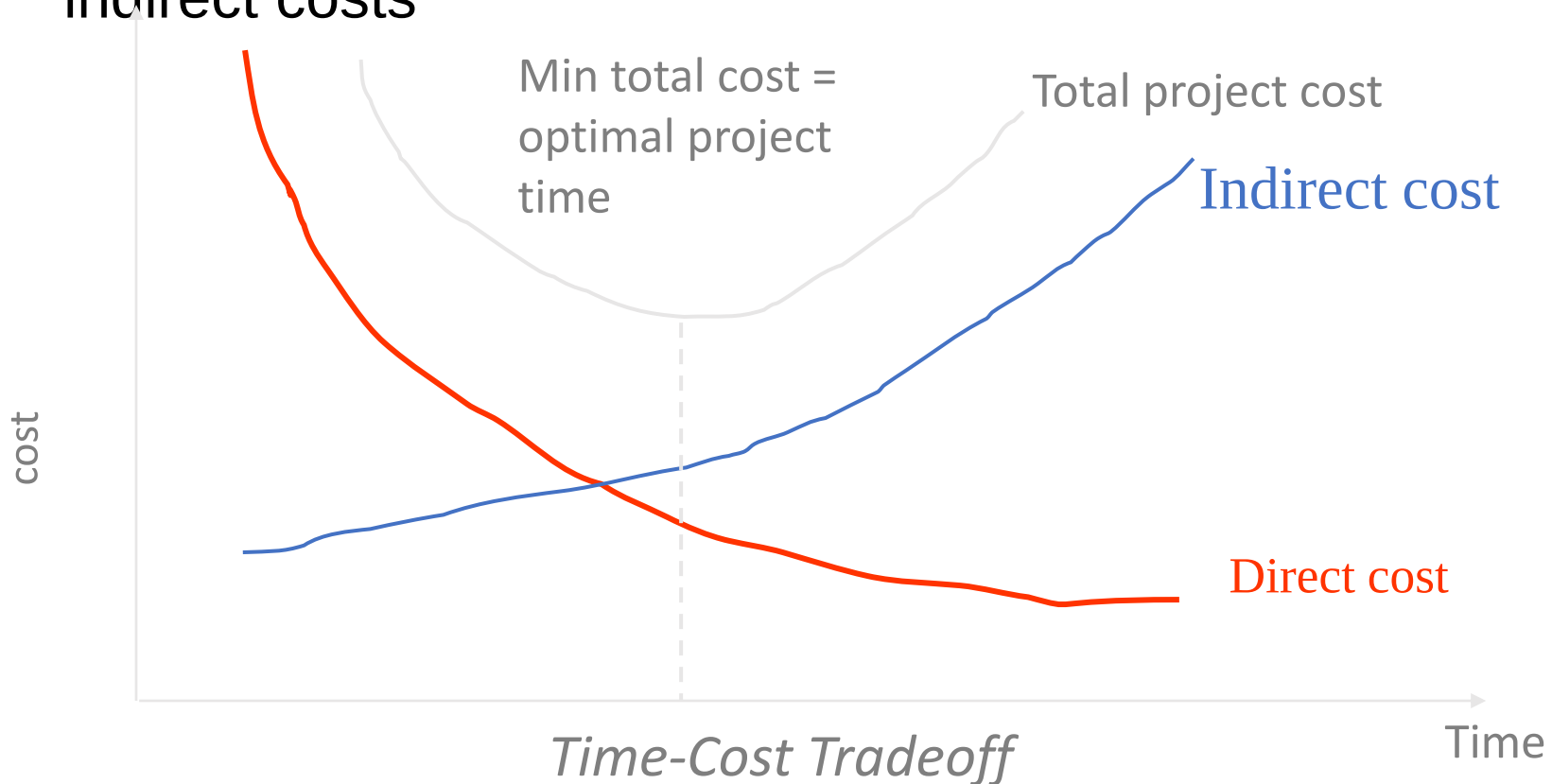
- Crashing
 - reducing project time by expending additional resources
- Crash time
 - an amount of time an activity is reduced
- Crash cost
 - cost of reducing activity time
- Goal
 - reduce project duration at minimum cost

Activity crashing

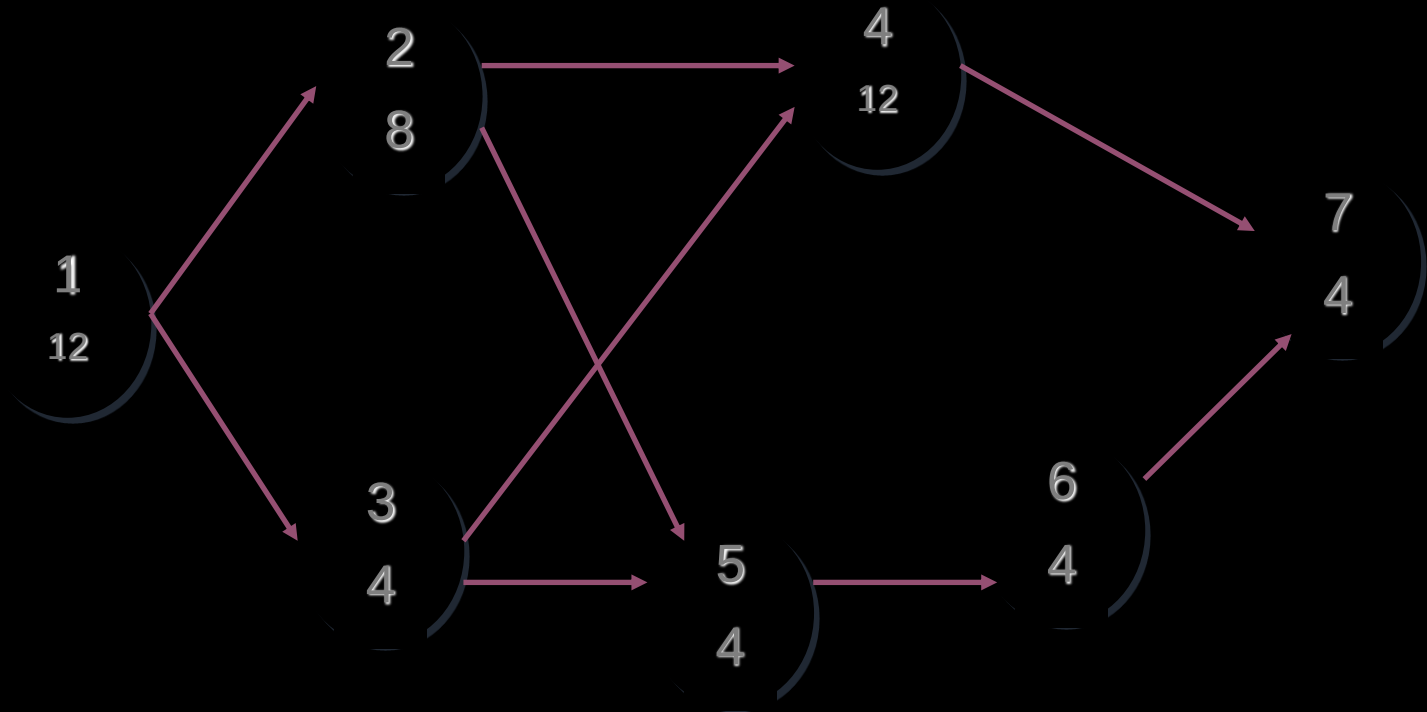


Time-Cost Relationship

- Crashing costs increase as project duration decreases
- Indirect costs increase as project duration increases
- Reduce project length if crashing costs are less than indirect costs



Project Crashing example

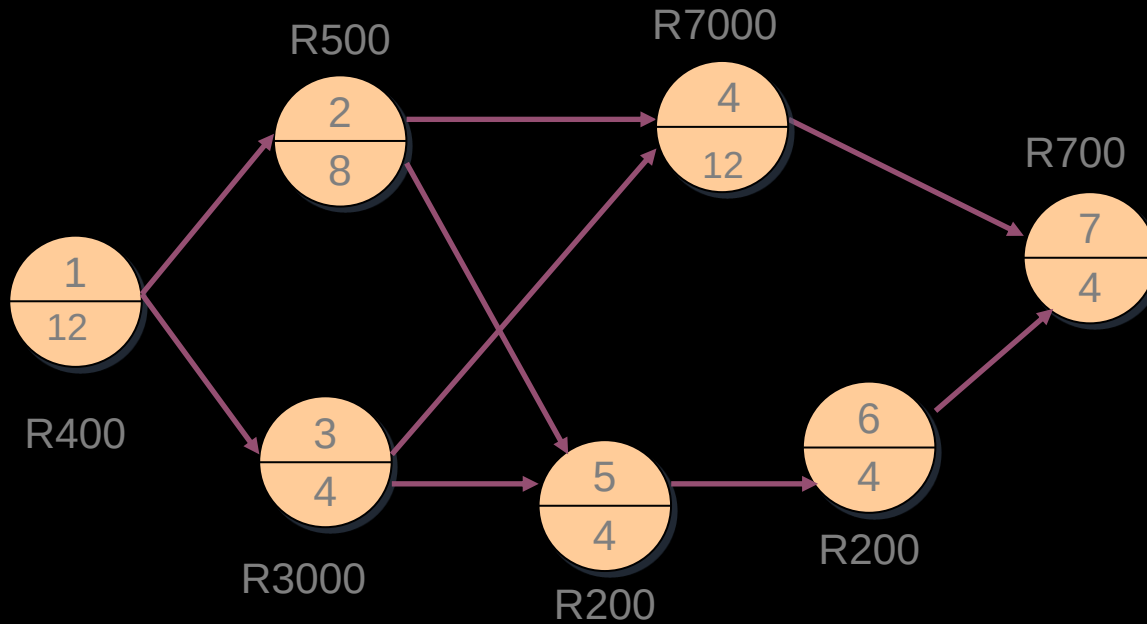


Time Cost data

Activity	Normal time	Normal cost Rs	Crash time	Crash cost Rs	Allowable crash time	slope
1	12	3000	7	5000	5	400
2	8	2000	5	3500	3	500
3	4	4000	3	7000	1	3000
4	12	50000	9	71000	3	7000
5	4	500	1	1100	3	200
6	4	500	1	1100	3	200
7	4	1500	3	22000	1	7000
		75000		110700		

Project duration = 36

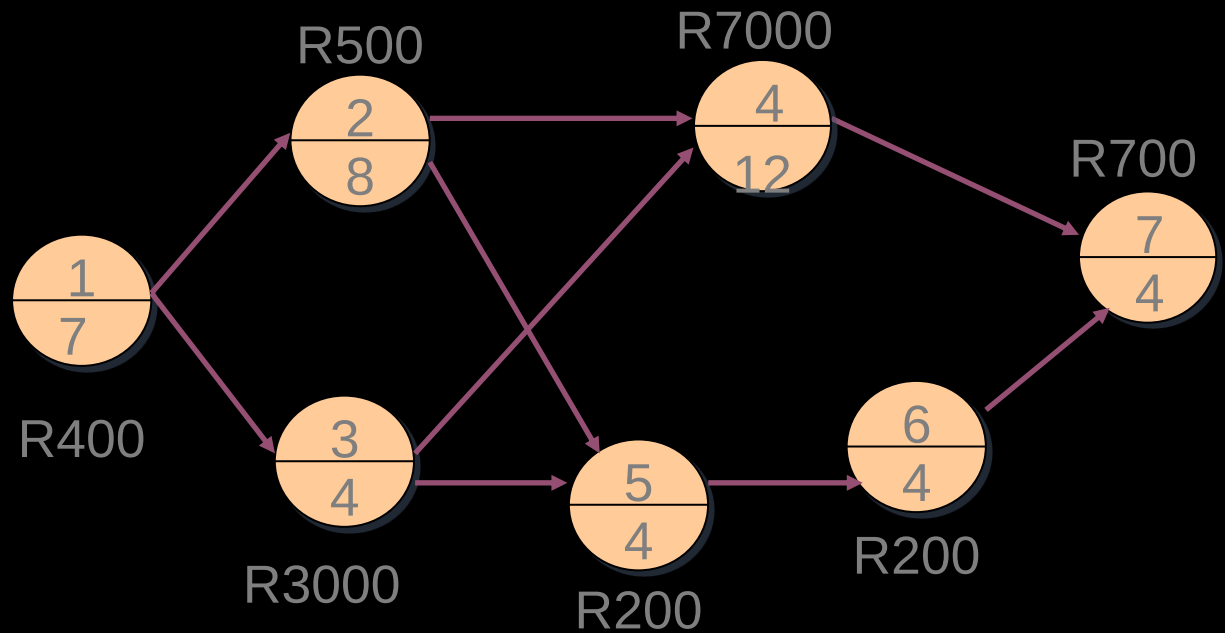
From.....



To.....

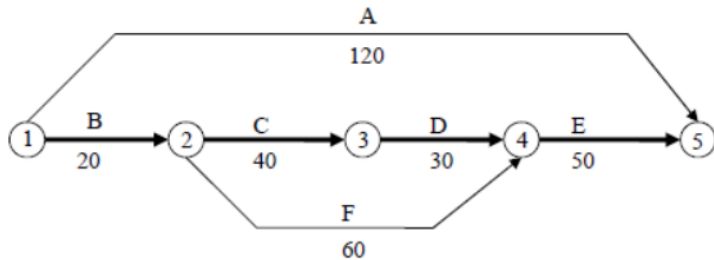
Project duration = 31

Additional cost = R2000



Question

The network and durations (in the table) given below shows the normal schedule for a project. You can decrease (crash) the durations at an additional expense. The Table given below summarizes the time-cost information for the activities. The owner wants you to finish the project in 110 days. Find the minimum possible cost for the project if you want to finish it on 110 days. (Assume that for each activity there is a single linear, continuous function between the crash duration and normal duration points).



Activity	Normal duration (days)	Crash Duration (days)	Normal Cost	Crash Cost
A	120	100	12000	14000
B	20	15	1800	2800
C	40	30	16000	22000
D	30	20	1400	2000
E	50	40	3600	4800
F	60	45	13500	18000

Answer

Assume that the duration-cost relationship for each activity is a single linear, continuous function between the crash duration and normal duration points. Using the normal duration (ND), crash duration (CD), normal cost (NC), and crash cost (CC), the crash cost slope for each activity can be determined as follows;

$$S_A = \frac{CC - NC}{ND - CD}$$

$$S_A = \frac{14000 - 12000}{120 - 100} = \$100 / day$$

$$S_B = \$200/day$$

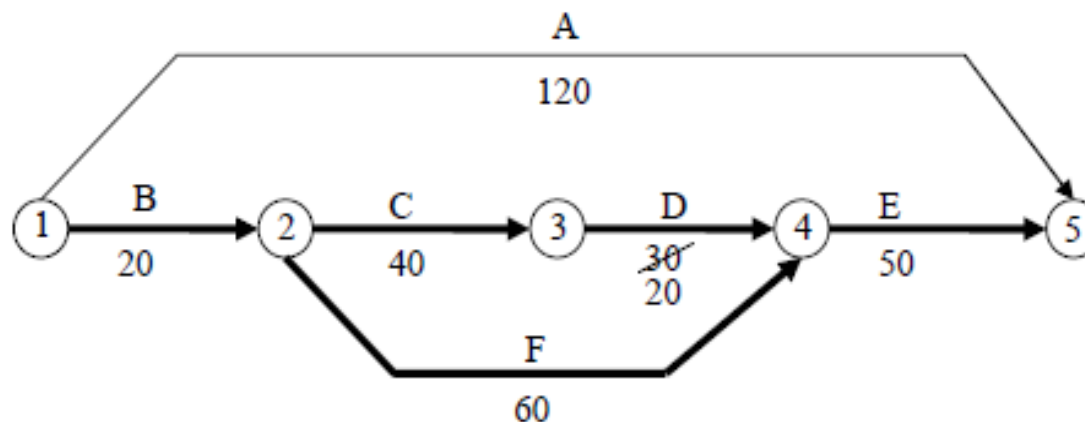
$$S_C = \$600/day$$

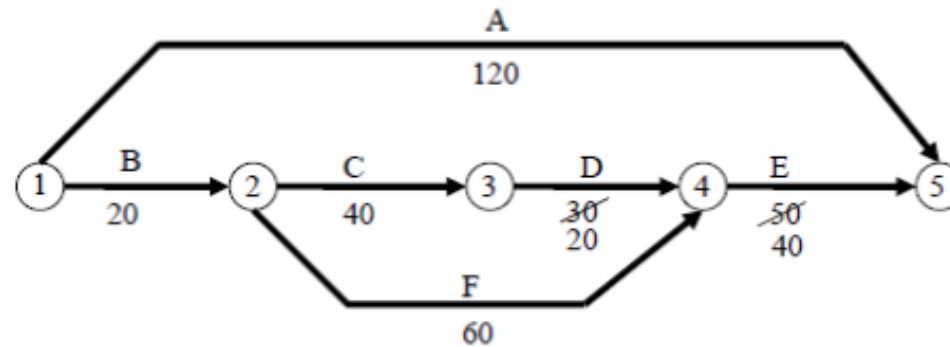
$$S_D = \$60/day$$

$$S_E = \$120/day$$

$$S_F = \$300/day$$

The normal cost for the project is the sum of a normal cost for each activity. The normal cost for the project is \$48300 and the normal duration is 140 days. The activity which should be crashed is the one on the critical path which will add the least amount to the overall project cost. This will be the activity with the flattest or least-cost slope. The duration can be reduced as long as the critical path is not changed or a new critical path is created. In addition, the activity duration cannot be less than the crash duration. $S_D = \$60/\text{day}$ (least-cost slope) Maximum of 10 days can be cut from this schedule by reducing the duration of activity D to the crash duration of 20 days.



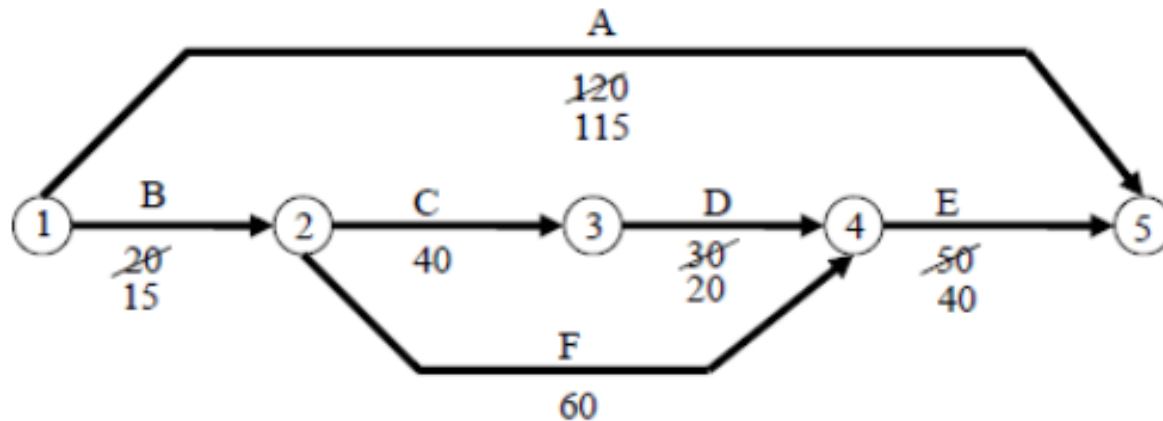


E by 10 days, the project duration is now 120 days and the total project cost is \$50100. There are now three critical paths (A, B-C-D-E, and B-F-E). The next stage of crashing requires a more thorough analysis since it is impossible to crash one path alone and achieve a reduction in the overall project duration. Activity A is paired with each of the other activities to determine which has the least overall cost slope for those activities which have remaining days to be crashed.

Activity A (\$100) + activity B (\$200)

Activity A (\$100) + activity C (\$600) + activity F (\$300)

The least-cost slope will be activity A + activity B for a cost increase of \$300 per day. Reducing the project duration by 5 days will add $5 \times 300 = \$1500$ dollar crashing cost and the total project cost would be \$51600. Activity B cannot be crashed any more.



Final step in crashing the project to 110 days would be accomplished by reducing the duration of activity A by a further 5 days to 110 days, reducing activity C by 5 days to 35 days, and reducing activity F by 5 days to 55 days. The combined cost slope for the simultaneous reduction of activity A, activity C, and activity F would be \$1000 per day. For 5 days of reduction this would be an additional \$5000 in total project cost. The total project cost for the crashed schedule to 110 days of duration would be \$56600

Benefits of CPM/PERT

- Useful at many stages of project management
- Mathematically simple
- Give critical path and slack time
- Provide project documentation
- Useful in monitoring costs

CPM/PERT can answer the following important questions:

- How long will the entire project take to be completed? What are the risks involved?
- Which are the critical activities or tasks in the project which could delay the entire project if they were not completed on time?
- Is the project on schedule, behind schedule or ahead of schedule?
- If the project must be finished earlier than planned, what is the best way to do this at the least cost?

Limitations to CPM/PERT

- Clearly defined, independent and stable activities
- Specified precedence relationships
- Over emphasis on critical paths
- Deterministic CPM model
- Activity time estimates are subjective and depend on judgment
- PERT assumes a beta distribution for these time estimates, but the actual distribution may be different
- PERT consistently underestimates the expected project completion time due to alternate paths becoming critical

To overcome the limitation, Monte Carlo simulations can be performed on the network to eliminate the optimistic bias

Computer Software for Project Management

- *Microsoft Project* (Microsoft Corp.)
- *MacProject* (Claris Corp.)
- *PowerProject* (ASTA Development Inc.)
- *Primavera Project Planner* (Primavera)
- *Project Scheduler* (Scitor Corp.)
- *Project Workbench* (ABT Corp.)